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Contraction-based Separation and Lifting for Solving the Max-Cut Problem

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Preface

The max-cut problem consists in partitioning the nodes of an undirected weighted graph into two sets such that the aggregate weight of the edges between these sets is maximized. This well-known combinatorial optimization problem is the reformulation, in graph theoretical terms, of the unconstrained 0/1 quadratic problem which aims at optimizing a quadratic objective function over the set of all 0/1 vectors of fixed dimension. In general, the max-cut problem is **NP**-hard, although selected special cases can be solved in polynomial time. It has a number of interesting applications such as the optimal design of very-large-scale-integration (VLSI) circuits or the study of minimum energy configurations of spin glasses—alloys of magnetic impurities diluted in a nonmagnetic metal—which is among the most investigated topics in the statistical physics literature.

The present book is concerned with finding provably optimal solutions of the max-cut problem as opposed to approximate solutions. To do so, we use the established and well-working branch-and-cut method, a generic solution technique whose performance for a given type of optimization problem is mainly determined by two key elements: Firstly, a close yet manageable approximation of the polyhedron associated with the problem. Secondly, efficient methods to solve the corresponding separation problem which is to decide for an arbitrary point in the ambient space whether or not it lies inside the polyhedron just mentioned.

For complete graphs, the max-cut problem and the associated cut polytope have been extensively studied over the last decades. Their counterparts on arbitrary graphs, in particular sparse ones, on the other hand, have received much less attention. Moreover, the transferability of methods from the complete to the sparse case is limited. This is mainly because the respective methods often require certain structures that are unlikely to be found in a sparse graph. A generic possibility to work around this problem is to make the graph artificially complete by adding zero-weighted edges. However, this technique is only effective in conjunction with an efficient way to exploit the original sparse structure. Otherwise, it will ultimately lead to the same computational complexity as the problem on the complete graph.

In this study, we investigate a new contraction-based separation approach for the max-cut problem that is primarily intended for problems on sparse graphs. The key idea is to contract edges based on their value in a given linear programming (LP) solution. In its simplest form, this technique presents an efficient way to separate so-called odd-cycle inequalities. In addition, we describe sophisticated methods to add missing edges to an already contracted graph as well as to compute suitable values to extend the corresponding LP solution accordingly. This allows us to apply solution techniques that were originally intended for problems on complete graphs and could not have been used on a sparse graph otherwise.

The book is structured as follows: In the first chapter, we introduce fundamental concepts, methods, and results from the fields of graph theory, complexity theory, linear and integer programming, as well as combinatorial optimization.

In Chapter 2 we precisely define the max-cut problem, including its reformulation in terms of quadratic optimization. We proceed with the description of two interesting applications coming from circuit layout design and statistical physics, respectively. Following a brief introduction to approximate solution techniques, we give a survey of the associated cut polytope and its facial structure. These combined results will help in devising a branch-and-cut algorithm later on. Finally, we outline the relevant literature and previous work on the max-cut problem.

Our key contribution, the new shrink separation approach, is presented in Chapter 3. Here, we elaborate on the single steps of the method and their respective underlying theory. Afterwards, we describe an actual realization of the shrink separation and point out its deviations from the theoretical conceptual design. Finally, we investigate some of the algorithm's numerical aspects.

Chapter 4 deals with the computational experiments that we carried out to test the performance of the shrink separation. After introducing the considered test instances, we specify the experiments' setup, including the hard- and software used, chosen parameters, and tested separation scenarios. We proceed by summarizing the results for the different classes of test instances before concluding with a case study that takes an in-depth look at a particularly interesting set of instances generated from real-world data.

The last chapter comprises a recapitulation of our contributions and findings in this study, followed by our conclusions and some suggestions regarding future research directions.

Finally, Appendices A and B contain the collective tables with detailed information on the characteristics of the test instances and the results of the computational experiments, respectively.

Author's Note

This book is a revised version of my doctoral thesis submitted and defended at the University of Heidelberg in 2011. Though the technical content is essentially the same as the submitted version, several aspects have been improved. In particular, Section 3.1.3 has been reworked and extended significantly. Also, the computational results have been updated, misprints and minor errors fixed, and the writing polished.

Acknowledgment

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Heidelberg, September 2011 Thorsten Bonato

Contents